RHYTHM: A MOTIVATION TO STUDY GEOMETRY

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Abstract

One of the main concerns of Mathematics teachers in Technical Schools of Engineering and Architecture is how to draw our students' attention and interest for the subject we teach. We have widely discussed this general worry in many occasions. The Research Group MAIC (Mathematics Applied to Civil Engineering) establishes among their aims to foster active participation among students in the process of learning - teaching Mathematics. The "Workshops to approach Mathematics" play a relevant role to achieve this aim. This new learning model is based on practical activities with students. This paper reports on one of them, a Workshop to introduce first year students in the study of proportions through a presentation of masterpieces of painting, architecture and art.

The theoretical foundations of this methodology are several definitions of proportion. Vitrubio, (I a. C) defines proportion as the commensurability of each of the members in a work and of all the members in the whole by a specific measure unit or module (*De architectura, book I).* In his *Teoria dell' architettura*, Bruno Taut ensures that the only aspect that acts in architecture is the sense of proportions. The essential aim of the theory of proportion is visual intentionality, which consists in creating an apparent order by the repetition of similar figures and their form intentionality, based, not on the forms themselves but on the rhythm among such forms. The similarity of figures in geometry and the analogy of planes and volumes in architecture emerge from this rhythm.

The activities carried out in the Workshops can be further developed with computer tools and software to obtain designs corresponding to the studied proportions.

This teaching experience will be presented from a multidisciplinary perspective by using an artistic approach to both architecture and civil engineering works. Among other aspects, the results show that, if study topics are in a direct relationship with the students' interest centers, motivation increases.

Keywords: Geometry, Innovation, Art, Proportion.

1 WORKSHOP ON RAFAEL DE LA-HOZ AND HIS OEUVRE

The course outline is as follows:

1.1 Cordoban proportion – the notion

1.2 Examples of cordoban rectangles in architecture

1.3 Rafael de La-Hoz Arderius and cordoban rectangles

1.1 Cordoban proportion – the notion

The Greek mathematician Euclid of Alexandria first established the principle of the "mean and extreme ratio", later known as the "golden ratio", "harmonious ratio", "golden section" or "golden rule" in Book II of his Elements of Geometry.

As the depositary and sole usufructee of the Euclidian legacy throughout the Middle Ages, Cordoba might reasonably be thought to have been the place par excellence where pre-Renaissance architecture made rational use of the golden ratio.

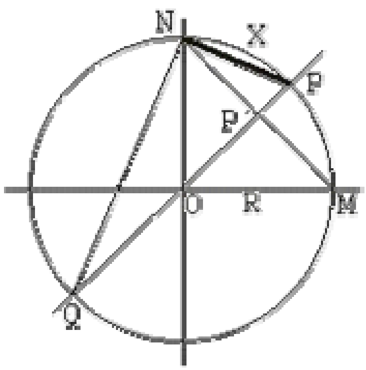
An anthropometric study of relief figures, sculptures and Roman mosaics in Cordoba, however, leads to the conclusion that the Romans who lived there were more inclined to represent their human figures in accordance with what is known as cordoban proportion or the cordoban constant,1,3.

Given that the golden number is the ratio between one side of a regular decagon and the radius of the circle in which it is inscribed, cordoban proportions might also logically be assumed to be built on geometric grounds.

And indeed, the respective value can be defined as the ratio between the radius of the circumference that delimits a regular octagon and one of its sides, which can be written as:

$$\frac{R}{L} = \frac{1}{\sqrt{2-\sqrt{2}}}$$

The quotient, c = 1,306562964…, is known as the cordoban number.



Geometric fundamentals of regular octagons

Take a circumference with radius R. If the first quadrant is bisected, the segment NP = X is the side of the regular octagon inscribed in that circumference.

Applying the Pythagorean theorem to triangle NOM yields:

$(MN)^2 = R^2 + R^2$, whereby MN = R√ 2

For reasons of symmetry, OP' = MN/2 (since NP'= MN/2 and OP' = P'N).

Given that QNP is a straight line, further to the cathetus theorem:

X /QP = P'P/ X

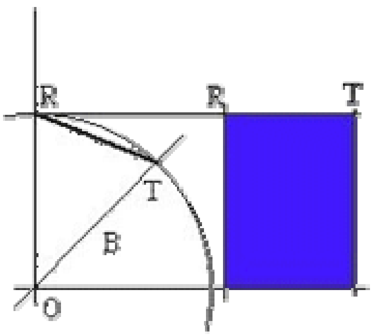
and hence: X^2 = QP. P'P = 2R (OP - OP')

in other words,

$$X = \sqrt{2-\sqrt{2}}$$

Assuming the radius of the circumference to be one (R = 1), the above expression can be rewritten as

$$\frac{R}{L} = \frac{1}{\sqrt{2-\sqrt{2}}}$$



With the foregoing in mind, a cordoban rectangle can be readily drawn by bisecting the first quadrant of any circumference. RT is one side of the rectangle and the radius of the circumference the other.

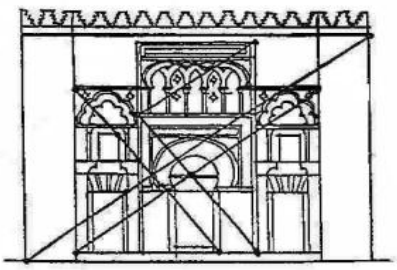
1.2 Cordoban rectangles in architecture

Cordoban rectangles first appeared in the city's Great Mosque and more specifically in the geometry of the Al-Hakam II gate, the Mih-rab façade, the floor plan and the interior arcade.

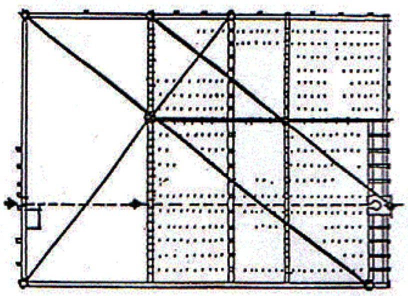
Fig 1 Cordoban vault over the ante-Mih-rab, Great Mosque of Cordoba

Fig 2 Al Hakam II gate, Great Mosque of Cordoba

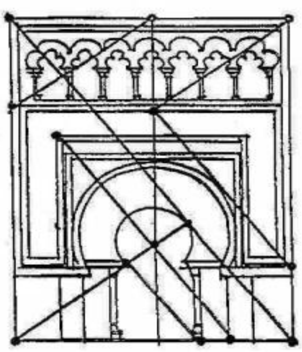
Expandable, modular and prefabricated mosque architecture, whose composition is based on Cordoban rectangles



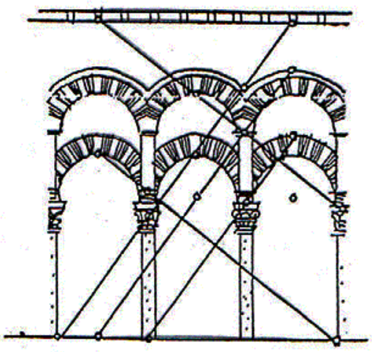
Fachada de Al-Hakam II



Planta de la Mezquita



Fachada del Mih-rab

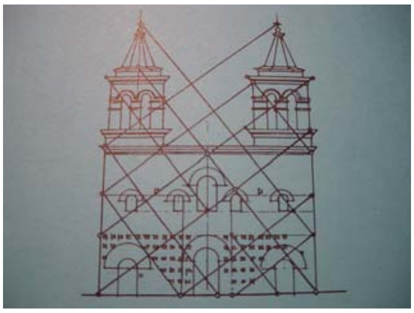


Arcadas de la Mezquita

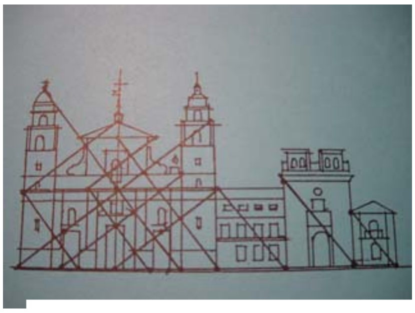
Cordoban rectangles were a basic element in caliphate and later architecture, where it can also be found.

In light of the difference between the cordoban experience and the Renaissance ideal, cordobans may in fact be thought to have developed their aesthetic values in accordance with their own ethnic characteristics. The Roman culture in Cordoba, for instance, generated reliefs, mosaics and sculptures whose authors, judging by the specimens found in the local archaeological museum, preferred other proportions.

This Cordoban idiosyncrasy is also visible in the architecture of other cultures and countries, such as:

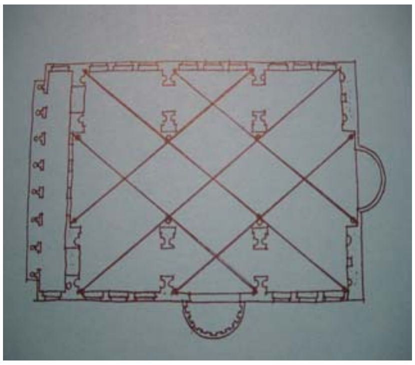


Jesuit Church at Cordoba, Argentina



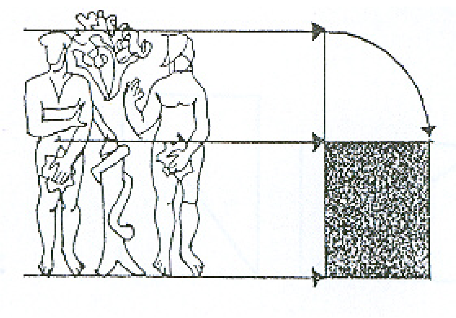
Primary Cathedral at Bogota

Arch of Triumph, Place de l'Etoile, Paris

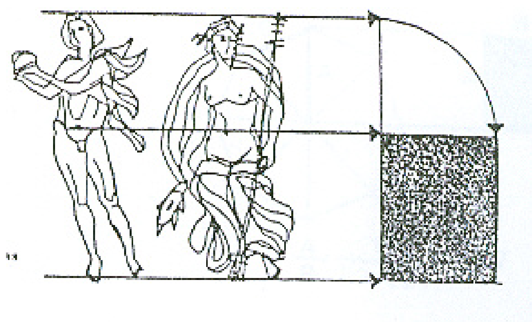


Massenzio Basilica

Examples of cordoban rectangles can also be found in other artistic expressions such as the Roman sculptures on exhibit at Cordoba's Archaeological Museum



Adam and Eve (Roman sarcophagus at Huerta de la Reina) Human proportions



Human figures in the Alcolea mosaic in the province of Cordoba

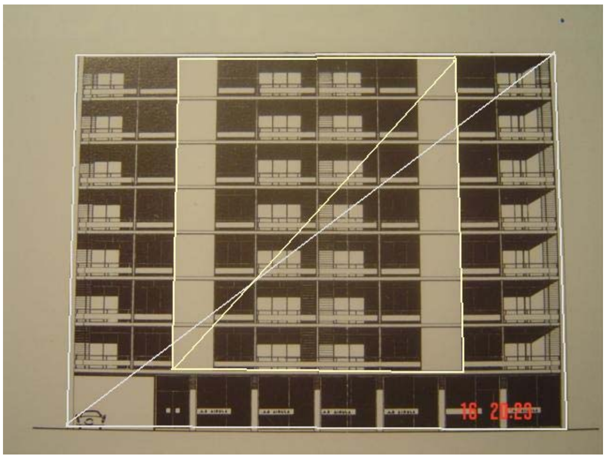
1.3 Rafael de La-Hoz Arderius and Cordoban rectangles

Rafael de la-Hoz was born at Cordoba on 9 October 1924 and died in Madrid on 13 June 2000. After spending his childhood in Cordoba, he graduated from the Madrid School of Architecture in 1951.

During his career, he not only studied cordoban rectangles in great depth, but applied the principle in many of the buildings he designed.

Further to the research conducted by Rafael de La-Hoz, examples of cordoban rectangles can be found in cordoban monuments and buildings.

Housing on Gran Capitán Street, Cordoba (1959)



The façade on this building clearly constitutes a cordoban rectangle and includes elements designed to the same ratio: windows, doors and structural modules.

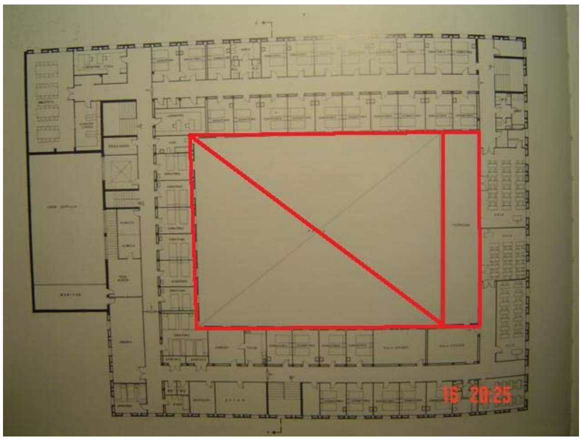
Elevation view of the building showing its

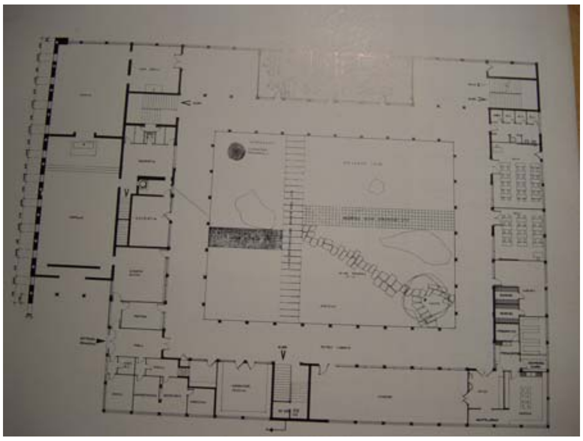
Cordoban rectangles

Salesian Convent, Cordoba (1959)

In this convent, the architect simplified the basic geometry, reducing the lines defining structure, masonry and the specifics of the monastic compound to a single system of abscissas and ordinates.

A series of coincidentally positioned rectangles, some cordoban, prevail in the structure of this compound, located on a line running north to south from the city to the mountains and visible from several vantage points.





Convent floor plan showing the proportional rectangles

Rafael de La-Hoz's oeuvre evolved in all respects. Initially guided by the values of early rationalism, from the late nineteen fifties onward when he undertook designs such as described above, he began to make more careful use of linear expression, showing a clear preference for orthogonal geometry.

His was a pursuit of rational functionalism where all manner of languages were feasible: relationships, scales, spatial hierarchy, openings, occlusions, tensions…

At the same time that he began to define space in rectangular terms, Rafael de La-Hoz discovered the hidden magic in metric relationships and the proportions of geometric, and particularly orthogonal, shapes. For the artist, who published his research on cordoban rectangles, the city was a repository of historical data on the golden ratio and “divine harmony”.

Logically, in light of the foregoing, the width, cells and church in the Salesian compound are proportionally related, the cloister and courtyard constitute a perfect square and the courtyard encompassing the cloister forms a harmoniously proportioned rectangle.

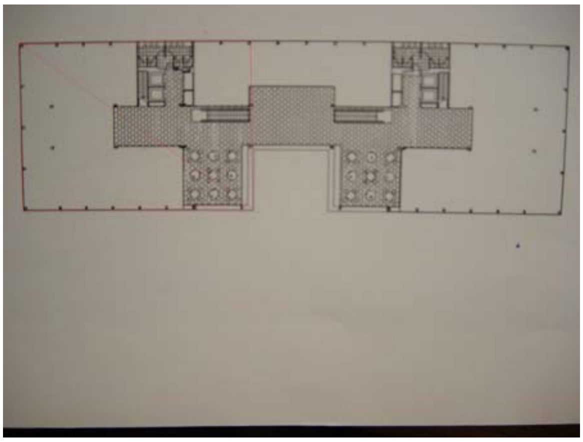
That the paths and gardens in this central unroofed area reinforce the geometry of squares and proportionally inter-related rectangles is equally unsurprising.

Mirasierra shopping mall (1986)

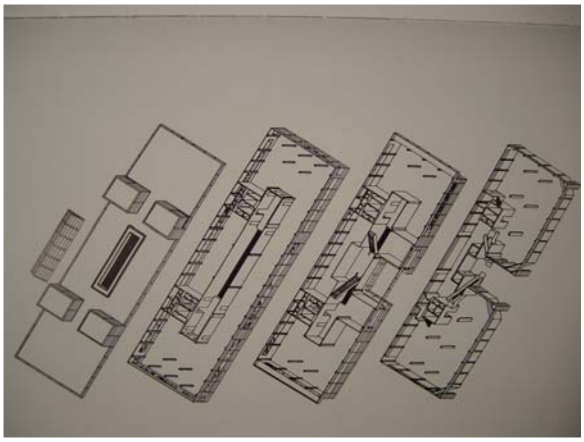
The building is structured around three modules, the outer two constituting an application of the cordoban rectangle.

They are connected by a central core that houses the main building services.

Some of the façades and windows may also be designed to cordoban rectangular proportions.

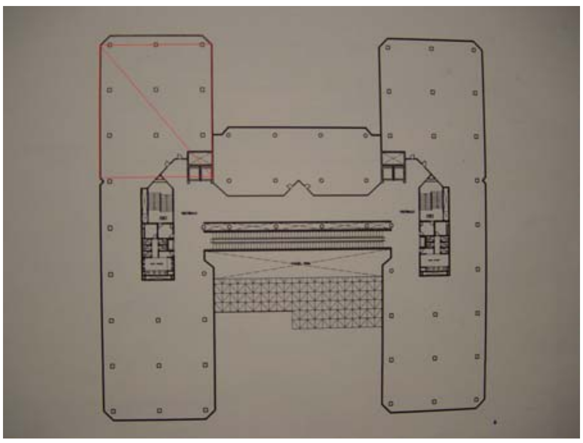


Floor plan showing cordoban rectangles



Drawings of the four storeys

Social Security Institute (“INSERSO”) headquarters, Madrid (1990)



This building is also structured around two identical modules on the sides connected by a differently designed inner corpus.

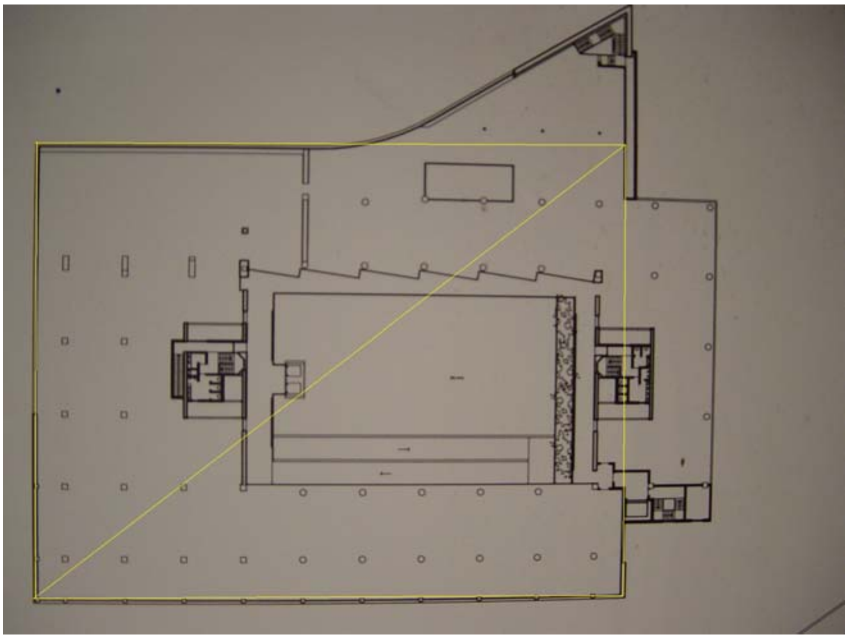
Four spaces that form part of the two outer structures are designed around Cordovan rectangles which, while concealed behind bevelled corners, are nonetheless visibly dimensioned to a ratio of 1,306.

Drawing of floor plan with Cordoban rectangles

Telefónica’s National Supervision and Organisation Centre, Aravaca, Madrid (1993)

The building’s composition is simple and transparent, with a floor plan designed to metric proportions and more specifically to the cordoban rectangle, albeit somewhat disguised by the insets and protrusions in the façade.

Courtyard



Ground storey floor plan, a cordoban rectangle

Further to the course syllabus proposed, the research on cordoban rectangles is based on the three stages of research: Acquisition of theoretical knowledge Analysis of applications in the namesake city Detection and confirmation of expansion of the use of this geometry in other areas of the Spanish empire

2 MATHEMATICAL CONTENT

The mathematical content covered in the course is as follows:

1. Isometries

Orthogonal transformations Translations

Isometries invariant points and invariant subspaces

Classification of R^2 and R^3 isometries

2. Similarities

Homoteties

R^2 and R^3 similarities

Students are introduced to the theory of proportion to enable them to apply the academic knowledge acquired, as follows:

The problem of harmony

The notion of proportion

Commensurable and incommensurable proportions

Polygons and lines

Spirals Stars

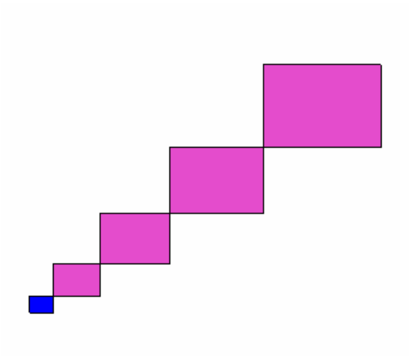
Growth and form

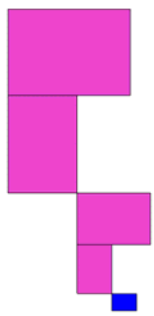
After analysing the work chosen, students engage in a practical exercise using Maple software, described below.

3 USE OF IT TOOLS

The use of IT tools is highly recommendable because of the many examples that can be readily generated and the graphic representation of the results.

The aim of the computer exercise is to enhance students' ability to handle abstract concepts and introduce them to iterative processes. It consists of two parts: Using Maple's "linalg" and plots libraries, students define similarity matrices to subsequently design their own figure.

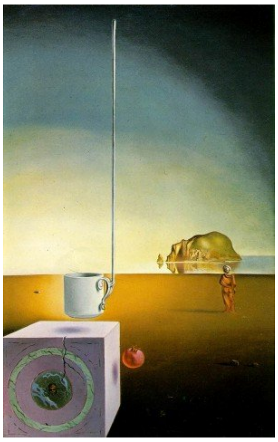
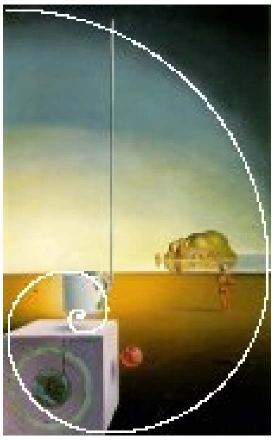
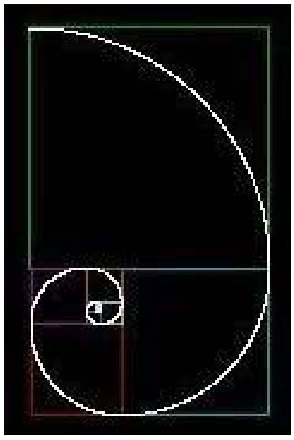


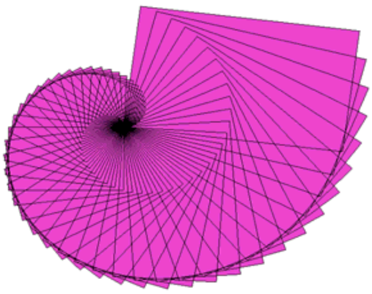


These are designs with cordoban rectangles.

This example is an exercises about de golden section.

Dalí (spiral confined within golden rectangles)



```
restart:with(plottools):with(plots):with(linalg):
Figuras:=table():
A:= table():B:= table():C:= table():G:= table():
phi:=(1+sqrt(5))/2:
n:= 50:
for i from 1 to n do
k :=2*Pi*i/n:
giro:=matrix([[cos(k),-sin(k)],[sin(k),cos(k)]]):
r:=i+1:homotecia:=matrix([[r,0],[0,r]]):
A[i+1]:=evalm(homotecia&*giro&*A[1]):B[i+1]:=evalm(homotecia&*giro&*B[1]):
C[i+1]:=evalm(homotecia&*giro&*C[1]):G[i+1]:=evalm(homotecia&*giro&*G[1]):
v1:=A[i+1][1]:v2:=A[i+1][2]:A[i+1]:=[v1,v2]:
v1:=B[i+1][1]:v2:=(B[i+1][2]):B[i+1]:=[v1,v2]:
v1:=(C[i+1][1]):v2:=(C[i+1][2]):C[i+1]:=[v1,v2]:
v1:=(G[i+1][1]):v2:=(G[i+1][2]):G[i+1]:=[v1,v2]:
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```
Figuras[i+1]:=polygon([A[i+1],B[i+1],C[i+1],G[i+1]],color=COLOR(RGB, 0.8960, 0.3000, 0.8000)):
end do:
display(seq(Figuras[i],i=1..n));
```

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